

CHAPTER 5

MATH AND SPORTS

5.1 SCORING AND STATISTICS

5.2 DISTANCES IN SPORTS

5.3 SPEED IN SPORTS

5.4 RACE TRACKS

CHAPTER SUMMARY

CHAPTER REVIEW

CHAPTER TEST

When we watch or participate in a sporting event, we seldom realize that mathematics has anything to do with what's going on. In this chapter, we will show that it does. We will show that math is alive in the world of sports. You will see that variables, equations, and formulas can be used in determining distances, analyzing statistics, investigating speeds, and more. For example, we will look at distances for passes or kicks in football, determine how many more hits a baseball player needs to bat 0.300, look at the speed of a sprinter in the 100 meter dash, and examine the distance on the outside lane of a 400 meter track.

Research Projects

1. What are conic sections? Discuss how they are produced, both geometrically and algebraically. What applications do they have? In particular, look at parabolas as the path traveled by projectiles such as shotputs, baseballs, basketballs, etc.
2. How are the following professional sports statistics computed?
 - a) Free throw champion (basketball)
 - b) Earned run average (baseball)
 - c) Slugging percentage (baseball)
 - d) Quarterback rating (football)
 - e) Long drive champion (golf)Explain and give examples of each.
3. What is the origin of the word algebra? Who were some of the founders of algebra and what were their contributions to the development of algebra?

Math Projects

1. From the dimensions of a regulation NBA basketball court determine the distance of a shot made from one corner of the court to the basket at the other end of the court. Next, calculate the distance of the shots made every 10 feet along the edge of the court until you reach the corner next to the basket. Plot your results on a rectangular coordinate system with distance from the basket on the x -axis and length of the shot on the y -axis. What can you say about the decrease in the length of shot as you move closer to the basket?
2. Records in running and swimming events have improved over the last 100 years. Are the runners and swimmers improving at comparable rates? To answer this question, examine the changes in the world records for the 100 m run and 100 m freestyle swim every 10 years since 1900.
 - a) Analyze the data graphically. Plot the record time for each 10 year period on a rectangular coordinate system (time on the y -axis and 10-year periods on the x -axis). Are the graphs similar?
 - b) Analyze the data numerically by determining percent decreases in the record times. How do the percent decreases compare between runners and swimmers?
 - c) Using the results of a) and b), what do you think the records will be in the year 2020? Justify your results.
 - d) Write a paragraph answering the question, “Are the runners and swimmers improving at comparable rates?”
3. Find the current world records for the major running events in track, 100 m, 200 m, 400 m, 800 m, 1600 m, 3000 m, 5000 m, and 10,000 m, for either men or women.
 - a) What is the average speed of each competitor in miles per hour (mph).
 - b) What is the percent decrease in the speed as the distance gets longer?
 - c) Using a) and b), estimate the record for a 15,000 m run. Justify your results.
 - d) If each of the record holders ran the mile at the speed you calculated for their record performance, what would be their time be in the mile run?
(Note: 1 mile \approx 1609.3 meters)

Section 5.1 Scoring and Statistics

Sports teams, coaches, athletes, and fans are frequently concerned about the statistics in their sport. A team may want to know how many games it needs to win to get above the “**500 mark**” (a winning percent of 50%). A coach may be concerned about her winning percentage during her career. A baseball player may want to determine how many hits are needed in the remaining four games to bat “**300**”. The sports fan may want to figure out how many free throws his favorite player needs to make in order to break the league free throw percentage record. These types of statistics are important in sports since they measure the success or the failure of a team, coach, or athlete. They provide much of the justification in contract negotiations or product endorsements. In this section, we will examine some of those situations and discover how math can help analyze sports statistics.

Example 1

Percentages are used frequently in sports to measure and compare the performances of athletes or teams.

- What is the batting average of a baseball player that has 64 base hits in 180 at-bats?
- What is the free-throw percentage (pct) for a basketball team that made 54 free-throws and missed 26 in a tournament?

Solution:

- A batting average in baseball is the ratio of hits to at-bats, represented as a decimal rounded to three decimal places. To change a ratio (fraction) to a decimal divide the numerator by the denominator.

$$64 \text{ hits in } 180 \text{ at-bats} = \frac{64}{180} = 0.3555 \approx 0.356$$

The baseball player has a 0.356 batting average, frequently listed as a batting average of 356. If we change that decimal to a percent by moving the decimal point two places to the right, we get 35.6%. This indicates that the player got a hit 35.6% of the time.

- To find the percent of free-throws made, convert the ratio of free-throws made to total attempts to a decimal and change the decimal to a percent.

Total attempts = free-throws made plus free-throws missed = $54 + 26 = 80$.

Ratio of free-throws made to total attempts: $\frac{54}{80} = 0.675 = 67.5\%$

The basketball team had a 67.5% free-throw percentage for the tournament.

Example 2

A professional baseball team has won 62 and lost 70 games. How many consecutive wins would bring them to the “500 mark”?

Solution:

Let x = the number of consecutive wins.

$62 + x$ = the number of wins

$62 + 70 + x$ = the total games played

$$\frac{62 + x}{132 + x} = \text{ratio of wins to total games}$$

Since the “500” mark is 50%, set that ratio equal to 0.50 and solve for x .

$$\frac{62 + x}{132 + x} = 0.50$$

$$62 + x = 66 + 0.50x$$

$$0.50x = 4$$

$$x = 8$$

By winning 8 consecutive games, the team would reach the “500” mark.

Example 3

A basketball player realizes that in order to be more competitive in salary negotiations, he needs to complete at least 88% of his free throw attempts by the end of the season. After 50 games, he has made 173 out of 202 attempted free throws. If in the last 42 games of the season, he expects to have 168 more free throws, how many of those does he have to make in order to reach the 88% free throw mark?

Solution: Let x = the number of free throws he needs to make out of 168.

$173 + x$ = the number of free throws made

$202 + 168$ = the number of free throws attempted

$$\frac{173 + x}{370} = \text{ratio of free throws made to attempted}$$

To find x set that ratio equal to 88% = 0.88 and solve.

$$\frac{173 + x}{370} = 0.88$$

$$173 + x = 325.6$$

$$x = 152.6 \approx 153$$

Thus, by making 153 out of his last 168 free throws, the player will reach the 88% mark.

Example 4

Professional golfers frequently drive a golf ball 300 or more yards but very few have an average distance of 300 or more yards for the year. For example, at one point in a season, Tiger Woods had an average of 293.4 yards out of 104 officially measured drives. What does he have to average on his next 40 measured drives to bring his average for the year up to 300 yards?

Solution: To find the average distance, find the total yards of all his drives divided by the number of drives.

Let x = the average of the next 40 drives.

$$\begin{aligned}\text{Total yards} &= 104 \text{ drives at } 293.4 \text{ yards plus } 40 \text{ drives at } x \text{ yards} \\ &= 104(293.4) + 40x = 30,513.6 + 40x\end{aligned}$$

$$\text{Number of drives} = 104 + 40 = 144$$

$$\text{Average} = \frac{\text{total yards}}{\text{number of drives}} = \frac{30,513.6 + 40x}{144}$$

Setting the average to 300 and solving for x , we get:

$$\begin{aligned}\frac{30,513.6 + 40x}{144} &= 300 \\ 30,513.6 + 40x &= 43,200 \\ 40x &= 12,686.4 \\ x &= 317.16\end{aligned}$$

Thus, Tiger Woods must average 317.16 yards on his next 40 measured drives to average 300 yards for the year.

Equations can also be used in analyzing scoring in sports. Suppose a place-kicker scored a school record of 17 points with field goals (3 points each) and extra points (1 point each). How many different ways could he have scored the 17 points? We could use a guessing process and determine the different ways to score 17 points, for example, 4 field goals and 5 extra points or 5 field goals and 2 extra points. However, if we are not careful in our analysis, we might miss some of the possible solutions. To help us be more accurate, we set up an equation to model the problem and then systematically find all solutions.

Let x = the number of field goals scored.
 y = the number of extra points scored.

Since each field goal is 3 points and each extra point is one point, we get the following.

$$3x + 1y = 17$$

There are some logical restrictions on x and y :

- both must be whole numbers;
- the maximum value for $x = 5$ since 6 field goals would be 18 points;
- and the maximum value for $y = 17$.

Thus, the solution to the problem becomes solving the equation $3x + 1y = 17$ where x and y are whole numbers, $0 \leq x \leq 5$, and $0 \leq y \leq 17$. There is no simple algebraic way to find all the solutions to such an equation. The best thing we can do is to set up a table for x and y and systematically find all solutions by assigning an integer value for x from 0 to 5 and calculating the value for y . Note: in the table the (3) and (1) next to the x and y were put there to remind us of the number of points for each x and y .

x (3 points)	y (1 point)
0	17
1	14
2	11
3	8
4	5
5	2

There are six ways in which the kicker could score 17 points.

Such an equation with integer coefficients and more than one integer solution is called a **Diophantine Equation** after the Greek mathematician, Diophantus (c. 250 A.D.).

Example 5

An archery target consists of five concentric circles as shown. The value for an arrow in each region starting from the inner circle is 9, 7, 5, 3, 1 points. In how many ways could five scoring arrows earn 29 points?

Solution: We can set up Diophantine equations to model the problem.

- Let a = the number of 9 point arrows.
- b = the number of 7 point arrows.
- c = the number of 5 point arrows.
- d = the number of 3 point arrows.
- e = the number of 1 point arrows.

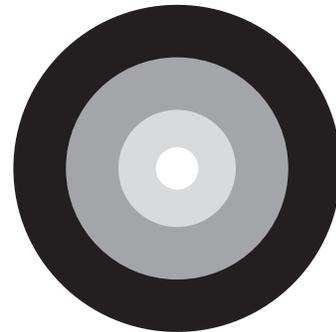
Since there are five scoring arrows,

$$a + b + c + d + e = 5.$$

Since the five arrows score 29 points,

$$9a + 7b + 5c + 3d + 1e = 29.$$

We can now set up a table of values and systematically find all possible values that simultaneously satisfy both equations. We can start with the largest value for a and determine the values for the other variables making sure we use whole numbers and a total of five arrows. The maximum value for a is 3, since when a reaches 4, $9a = 36$, which is over the 29 total points.



a (9)	b (7)	c (5)	d (3)	e (1)
3	0	0	0	2
2	0	2	0	1
2	1	0	1	1
1	2	1	0	1
1	2	0	2	0
1	1	2	1	0

a (9)	b (7)	c (5)	d (3)	e (1)
1	0	4	0	0
0	4	0	0	1
0	3	1	1	0
0	2	3	0	0

There are ten ways to attain a score of 29 with five arrows.

You can see from the last example that even with the use of equations the solution to the problem required that you think logically and be systematic in your approach.

The examples in this section have shown you that math can be used in analyzing sport stats and scoring. The problems that follow will present more situations in which you can apply those techniques.

§5.1 Explain → Apply → Explore

Explain

1. How are ratios converted to percents?
2. How are percents converted to ratios?
3. What is the “500” mark in sports statistics?
4. How is a losing percentage determined if a team wins x games and loses y games?
5. How is a winning percentage determined if a team wins x games and loses y games?
6. If a baseball player has a batting average of .315, what does that mean?
7. What is a Diophantine Equation?

Apply

8. In the 2004 NFL season, Daunte Culpepper of the Minnesota Vikings completed 379 out of 548 passes. What was his passing percentage?
9. Famous professional football coach Vince Lombardi as coach of the Green Bay Packers won 141, lost 39, and tied 4 games. What was his winning percentage? What percent of his game ended in ties? (Note: Treat the four ties as two additional wins and two additional losses.)
10. In the 2004 LPGA (Ladies Professional Golf Association) season, Annika Sorenstam had 38 out of 46 rounds under par while Lorena Ochoa has 52 out of 70 rounds under par. Which golfer had a better round-under-par percentage?
11. In the 2004 Australian Open, Venus Williams made 34 out of 52 of her first serves while her opponent Anna Smashnova made 24 out of 38. Which tennis player had a better first-serve percentage?
12. If a tennis player has won 15 out of 20 matches, how many consecutive wins would bring her to a winning percent of 80%.
13. A soccer goal keeper stopped 75.7% of the 181 shots made on the goal. How many consecutive stops would bring his percent up to 80%?
14. A softball team has won 3 and loss 8 games. How many consecutive wins would bring them to the “500-mark”?

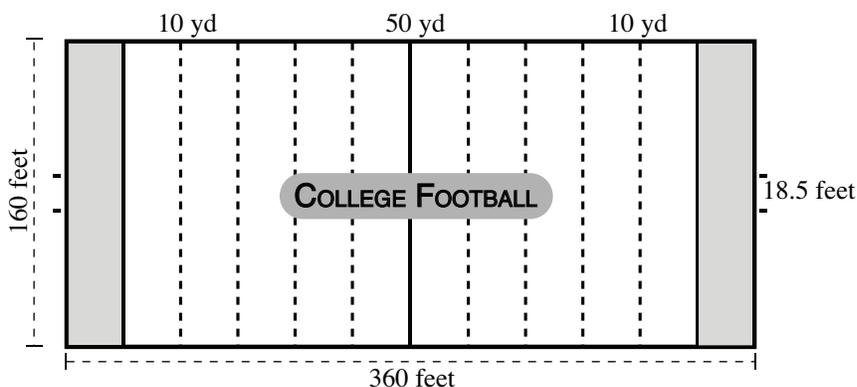
15. If a National Hockey League team, has a record of 25 wins, 36 losses, and 8 ties, how many consecutive wins would bring them to a 50% winning percentage? (Note: Treat the eight ties as four additional wins and four additional losses.)
16. A discus thrower has averaged 225 feet on his best throw for the first three meets of the season. What must he average on the best throw for the next six meets be to have an average of 230 ft for the season?
17. A Professional Bowlers Association senior bowler is averaging 210 for five tournaments. What must she average on the next 7 tournaments to set a personal scoring average of 227 for the season?

Explore

18. How many ways could you score 15 points in a basketball game? Set up a Diophantine equation using possibilities of 1-point, 2-point, and 3-point baskets.
19. In the game of horseshoe pitching, a horseshoe can score 3 points, 2 points, or 1 point for a player depending on its position relative to the stake. In how many way can a horse-shoe player reach a score of 11 points? Set up a Diophantine equation using possibilities of 1-point, 2-point, and 3-point scores.
20. In a football game without a safety, you can score 3-points with a field goal, 7-points with a touchdown and extra point, 6-points with a touchdown and no extra-point, and 8-points with a touchdown and a 2-point conversion. Under those conditions, how many different ways can 21 points be scored in a football game?
21. In a track meet where the first place finisher is awarded 5-points, second place, 3-points, third place, 1-point, and zero points for fourth place or lower, how many ways can a team score 21 points in seven events? Assume that a team has only one participant in each event.
22. In a boxing match a boxer is awarded 7, 8, or 9 points per round, based on his performance in the round. In a 10-round boxing match, how many ways can a boxer score 82 points?

Section 5.2 Distances in Sports

In athletic competitions, we are frequently concerned about distances. We might ask, “How far did he hit that home run?”, “What was the distance on that winning field goal?”, or “How long was that punt return?” In newspaper and magazine articles, we may have read that Barry Bonds hit a 410 ft home run, Jason Elam made a 63 yard field goal, John Daly had a 320 yard drive, Curtis Martin ran for 178 yards. We may be in awe of these accomplishments and never stop to really analyze them. In this section, we use some math to take a closer look at some of the distances in sports.



In football, the distance for punts, passes, and kicks is measured along the ground from the yard marker where the ball is put into play to the yard marker at the end of a play. It does not take into account how far the play starts behind the line of scrimmage, the width of the end zone, or the fact that the play may go at angles other than straight up the field. Because of this, distances for a given play in football statistics are usually shorter than the actual distance of the play.

Example 1

A punt returner catches a punt on the 17-yard line and returns it to the 35-yard line on the same side of the 50-yard line. How long of a return is he credited for?

Solution: Since the play started and ended on the same side of the 50-yard line, the runners distance can be found by simply subtracting the values of the two yardage markers.

$$35 - 17 = 18 \text{ yards}$$

If the punt return did not go straight up the field, the actual distance the player ran is more than 18 yards. However, the player is credited with only an 18 yard return.

Example 2

A football player returns the kick-off from the 17-yard line to the 35-yard line on the other side of the 50-yard line, how long is the run?

Solution: Since the run went across the 50-yard line, we determine the distance from the start of the run (the 17-yard line) to the 50-yard line by subtracting 17 from 50 and find the distance from the 50-yard line to the end of the run (the 35-yard line) by subtracting 35 from 50. Thus, the total yardage is given by the following.

$$\begin{array}{r}
 50 - 17 = 33 \\
 50 - 35 = 15 \\
 \hline
 \text{total} = 48 \text{ yards}
 \end{array}$$

That seems to be a lot of work for a calculation that is done frequently in football. Let's look a little closer at this problem and use algebra to help us get a "better" way to obtain the solution.

Let: x = the yardage mark on one side of the 50-yard line
 y = the yardage mark on the other side of the 50-yard line
 $50 - x$ = distance from x to the 50 yard line
 $50 - y$ = distance from the 50 yard line to y

$$\begin{aligned}
 \text{Total yardage} &= (50 - x) + (50 - y) = 100 - x - y \\
 &= 100 - (x + y)
 \end{aligned}$$

Thus, to find the distance from the yardage marker on one side of the 50-yard line to the a marker on the other side of the 50-yard line, simply add the values of the two yardage markers and subtract from 100. Thus, in Example 2, the distance of the run was:

$$100 - (x + y) = 100 - (17 + 35) = 100 - 52 = 48 \text{ yards}$$

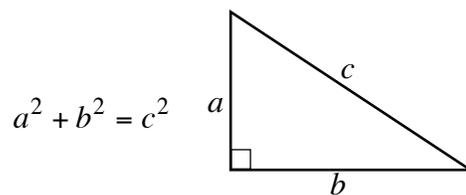
Example 3

How long is a pass that went from 24-yard line to the 8-yard line on other side of the 50-yard line?

Solution: Distance = $100 - (24 + 8) = 100 - 32 = 68$ yards

The Pythagorean Theorem

There are many instances when distances in sports can be determined using the **Pythagorean Theorem**. This theorem states for a right triangle with legs a and b and hypotenuse c ,

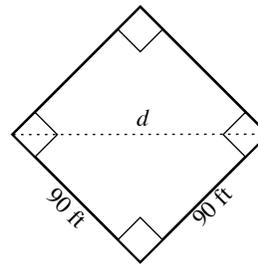


Example 4

How long is the throw from third base to first base on a professional baseball diamond where the bases are 90 feet apart?

Solution: A right triangle is formed by the third base line and the first base line with the throw from third to first is the hypotenuse of the right triangle, If d represents the distance from third base to first base, using the Pythagorean Theorem, we get the following.

$$\begin{aligned}
 a^2 + b^2 &= d^2 \\
 90^2 + 90^2 &= d^2 \\
 8100 + 8100 &= d^2 \\
 16200 &= d^2 \\
 \sqrt{16200} &= \sqrt{d^2} \\
 127.3 &\approx d
 \end{aligned}$$



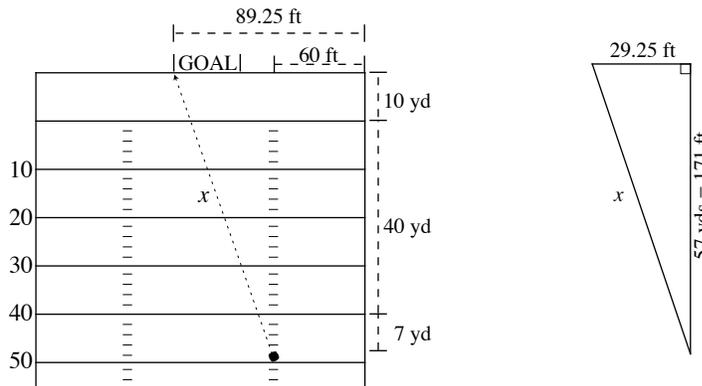
Example 5

If a college football player kicks a field-goal when the ball was put in play 40 yards from the goal line, what is the maximum length of a field goal as measured along the ground?

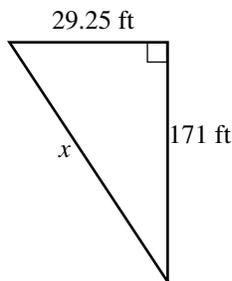
Solution: To find the maximum length, we have to take the following into consideration:

1. Field goals are kicked from a point 7 yards behind the scrimmage line.
2. The field goal is kicked from the right hash mark and cleared the left goal post.
3. From the diagram of the football field below, we see that:
 - the goal is 10 yards deep in the end zone
 - the hash mark is 60 ft from the side line
 - the left goal post is 89.25 ft from the side line.

Let x = the distance to the farthest goal post along the ground.



The ball was kicked 7 yards behind the scrimmage line and there are 40 yards to the goal line and 10 yards more to the end line where the goal post is located. Thus, the perpendicular distance from the point which the ball was kicked to the end line is $7 + 40 + 10 = 57$ yards or 171 feet. The distance along the end line from the left goal post to the perpendicular is $89.25 \text{ ft} - 60 \text{ ft} = 29.25 \text{ ft}$. The right triangle formed can be solved using Pythagorean Theorem.



$$x^2 = 29.25^2 + 171^2$$

$$x^2 = 855.5625 + 29241$$

$$x^2 = 30096.5625$$

$$\sqrt{x^2} = \sqrt{30096.5625}$$

$$x \approx 173.5 \text{ ft} \approx 57.8 \text{ yd}$$

Thus, even though the ball was kicked with a 40-yard scrimmage line, the maximum distance of the kick as measured along the ground is about 57.8 yards.

In the exercises at the end of the section, you will see that distances in other sports such as golf, tennis, baseball, and basketball can also be found using the techniques shown in this section.

§5.2 Explain → Apply → Explore

Explain

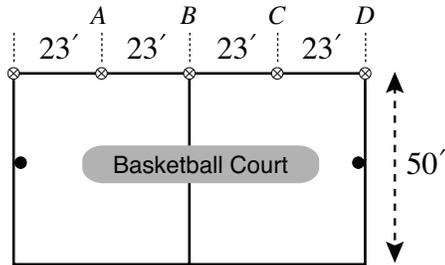
1. Explain two ways to find the distance of a football play that begins at a marker on one side of the 50-yard line and ends at a marker on the other side of the 50-yard line.
2. What is the Pythagorean Theorem?
3. When using the Pythagorean Theorem where must you place the length of the longest side of the triangle?

Apply

4. How much of a gain is a football run that starts on the 22-yard line and ends on the opposite 37 yard-line?
5. How long is a punt in football that travels from the 15 yard-line to the opposite 39-yard line?
6. In women's softball the bases are 60 feet apart. How long is the throw from home plate to second base?
7. If a Little League T-ball diamond has bases that are 50 feet apart, how long is the throw from first base to third base?
8. A tennis court is a rectangle with a width of 27 feet and a length of 78 feet. What is the distance from one corner of the court to the opposite corner of the court?
9. A doubles tennis court is a rectangle with a width of 36 feet and a length of 78 feet. What is the distance from one corner of the court to the opposite corner of the court?
10. In a college football field, what is the distance from a corner of one end zone to the opposite corner in the other end zone?
11. In a college football field, what is the distance from a corner at one goal line to the opposite corner at the other goal line?
12. A college quarterback throws a pass from the left hash-mark at his 30-yard line of scrimmage near the goal line to the right hand corner of the end zone. Even though he is credited for a 30 yard touchdown pass, what was the actual distance of the pass?
13. The "Green Monster" in Fenway Park in Boston is a wall that runs perpendicular to the

3rd base line 310 ft from home plate. The wall is 37.5 ft high and 240 ft long. What is the distance from home plate down the third base line measured to the top of the wall?

14. In Problem 13, what is the distance from home plate to the end of the Green Monster (240 ft down the wall) as measured along the ground? as measured to the top of the wall?



15. In the basketball court shown in the figure, how long is a shot made from point A and from point B to the basket on the left side of the court?
16. In the basketball court shown in the figure, how long is a shot made from point C and from point D to the basket on the left side of the court?

Explore

17. A college football place-kicker kicks a field goal from the left hash mark at the 30-yard line and is given credit for a 40 yard field goal. If we take into account the width of the end zone, the width of the goal post, and use the dimensions of a standard football field, what are the minimum and maximum distances of the field goal as measured along the ground?
18. If the place-kicker in Problem 17 kicked the field goal from the right hash mark at the 35-yard line, determine the maximum and minimum distance.
19. A punter kicks a football from the right hash mark at the 8 yard line that goes out of bounds at the left 36 yard marker on the other side of the 50 yard line. How far did the punt travel as measured along the ground?
20. A golfer has hit the ball in the rough 40 yards from the white marker in the fairway. The white marker indicates that the distance to the center of the green is 150 yards. Assuming that the ball lies in a line that is perpendicular to the line from the white marker to the center of the green, how long of a shot does the golfer have?
21. A golfer has hit the ball in the sand trap 30 yards from the red marker in the fairway. The red marker indicates that the distance to the center of the green is 100 yards. Assuming that the ball lies in a line that is perpendicular to the line from the red marker to the center of the green, how long of a shot does the golfer have?
22. An archer is shooting at three targets placed in a straight line 40 feet apart. The archer is positioned 90 feet perpendicular to the middle target. What is his distance to each of the three targets?
23. An archer is shooting at five targets placed in a straight line 20 feet apart. The archer is positioned 100 feet perpendicular to the middle target. What is her distance to each of the five targets?

Section 5.3 Speed in Sports

Speed is the name of the game in many athletic events. We might hear the following being asked. “How fast does he run 40 yards?” “What was her speed in the Indy 500 trials?” “How fast was that quarter-horse?” “If he kept up that speed, what would his mile time be?” In this section, we will see that algebra can help us answer such questions and analyze speed in sports.

Converting Units That Measure Speed

A speed may be given in many different units, such as, miles per hour (mph), feet per second (ft/s), kilometers per hour (km/hr), or meters per second (m/s). A speed measures a change in distance traveled over a period of time. Thus, if d = the distance traveled, t = the time to travel that distance, and r = the average rate of speed, then the relationship between them is given by the following formulas.

$$d = rt \quad \text{or} \quad r = \frac{d}{t}$$

It is important when determining the rate of speed to be aware of the units that are used. You may have to convert the time or distance to other units. One way to accomplish changing units is to set up and solve proportions using established conversion facts for distances and times. The facts listed in the charts below will be used throughout the section.

Time	Distance
1 min = 60 sec	1 yd = 3 ft
1 hr = 60 min = 3600 sec	1 mi = 1760 yd = 5280 ft
1 day = 24 hr	1 km = 0.621 mi
	1 mi = 1609.3 m
	1 furlong = 220 yd = 660 ft

Example 1

A football player runs 40 yards in 4.2 seconds, what is his speed

- in feet per second and
- miles per hour?

Solution:

- We are given the distance in yards but we need the distance in feet to represent the rate in feet per second. To convert yards to feet we can set up and solve a proportion comparing yards to feet. Using the conversion fact, 1 yd = 3 ft, we get:

d = the distance in feet

$$\frac{1 \text{ yd}}{3 \text{ ft}} = \frac{40 \text{ yd}}{d \text{ ft}}$$

$$\frac{1}{3} = \frac{40}{d}$$

$$1d = 3(40)$$

$$d = 120 \text{ ft}$$

Thus, the football player ran a distance of 120 ft ($d=120$) in 4.2 seconds ($t = 4.2$)

and the rate (r) is $r = \frac{d}{t} = \frac{120 \text{ ft}}{4.2 \text{ sec}} \approx 28.6 \text{ ft/sec}$.

- (b) To find the rate in miles per hour, we can change the distance, 120 ft, into miles and the time, 4.4 seconds, into hours. As before, we do this by setting up and solving proportions using the conversion facts, 1 mi = 5280 ft and 1 hr = 3600 sec.

Distance: Let d = the distance in miles

$$\frac{1 \text{ mile}}{5280 \text{ ft}} = \frac{d \text{ miles}}{120 \text{ ft}}$$

$$\frac{1}{5280} = \frac{d}{120}$$

$$5280d = 120$$

$$d = 0.022727 \text{ mi}$$

Time: Let t = the time in hours

$$\frac{1 \text{ hr}}{3600 \text{ sec}} = \frac{t \text{ hr}}{4.4 \text{ sec}}$$

$$\frac{1}{3600} = \frac{t}{4.4}$$

$$3600t = 4.4$$

$$t \approx 0.001222 \text{ hr}$$

The rate (r) is $r = \frac{d}{t} = \frac{0.022727 \text{ mi}}{0.001222 \text{ hr}} \approx 18.6 \text{ mph}$.

Using the basic formula for determining the average rate (speed), $r = d/t$, and proportions to convert from one unit to another, we can now analyze and compare speeds of different events. You should be aware that frequently the time of an event will be written in that standard form hours:minutes:seconds, for example, 5:3:45.12 means 5 hr, 3 min, 45.12 sec and 24:08.5 means 24 min, 8.5 sec.

Example 2

Hicham el Guerrouj of Morocco holds the world track records for both the mile and metric mile (1500m). On July 17, 1999 he set the mile record of 3:43.13 and on July 14, 1998, he set the 1500 m record of 3:26.00. Which record was run at a faster speed in miles per hour?

Solution:

To find the rate in miles per hour, write the distances in miles and the times in hours.

(a) The time for the mile run is 3:43.13.

Convert time to hours.

$$3:43.13 = 3 \text{ min } 43.13 \text{ sec} = 3 + \frac{43.13}{60} \text{ min} = 3.718833 \text{ min}$$

(Note: seconds are changed to minutes by dividing by 60.)

$$\frac{1 \text{ hr}}{60 \text{ min}} = \frac{t \text{ hr}}{3.718833 \text{ min}}$$

$$60t = 3.71883$$

$$t \approx 0.06198 \text{ hr}$$

The distance is 1 mile so

$$r = \frac{d}{t} = \frac{1 \text{ mi}}{0.06198 \text{ hr}} \approx 16.13 \text{ mph.}$$

(b) The time for the 1500 m run is 3:26.00

Convert time to hours.

$$3:26.00 = 3 \text{ min } 26 \text{ sec} = 3 + \frac{26}{60} \text{ min} = 3.433333 \text{ min}$$

$$\frac{1 \text{ hr}}{60 \text{ min}} = \frac{t \text{ hr}}{3.433333 \text{ min}}$$

$$60t = 3.433333$$

$$t \approx 0.05722 \text{ hr}$$

The distance is 1500 m so convert meters to miles.

$$\frac{1 \text{ mi}}{1609.3 \text{ m}} = \frac{d \text{ mi}}{1500 \text{ m}}$$

$$1609.3d = 1500$$

$$d = 0.93206 \text{ mi}$$

Thus, the rate is

$$r = \frac{d}{t} = \frac{0.93206 \text{ mi}}{0.05122 \text{ hr}} \approx 16.29 \text{ mph.}$$

The 1500 m race was run at a faster rate than the mile run.

Example 3

In Example 2, if Hicham el Guerrouj ran the mile at the same pace (rate) as the 1500 m, what would be his time for the mile?

Solution: To find the time it would take to run the mile at the pace of the 1500 m we would use the fact that $t = d/r$. The distance (d) is 1 mile and as determined in Example 2, the

rate is 16.29 mph.

$$t = \frac{d}{r} = \frac{1 \text{ mi}}{16.29 \text{ mph}} \approx 0.061395 \text{ hr}$$

The time in minutes and seconds can be calculated as follows.

$$\begin{aligned} t &= 0.061395 \text{ hr} = 0.061395(60 \text{ min}) = 3.6837 \text{ min} = 3 \text{ min} + 0.6837 \text{ min} \\ &= 3 \text{ min} + 0.6837(60 \text{ sec}) = 3 \text{ min} + 41.02 \text{ sec} = 3:41.02. \end{aligned}$$

Using the techniques presented in this section, you will be able to analyze speed in sports such as, track and field, bicycling, auto racing, horse racing, swimming, etc.

§5.3 Explain → Apply → Explore

Explain

1. What formula is used to determine the average rate (speed)? What do the variables represent?
2. How do you convert hours into seconds?
3. How do you convert seconds into hours?
4. How do you use proportions to convert kilometers to miles?
5. How do you use proportions to convert miles to kilometers?

Apply

In Problems 6 – 17, find the average speed in miles per hour (mph) for each feat.

6. The fastest pitch in softball in the 1996 Olympics (the first year that softball was an Olympic sport) was 118 kilometers per hour.
7. In 2004, Lance Armstrong (USA) won the 19th stage of the Tour de France bicycling at 49.4 kilometers per hour.
8. Lenny from Malibu set a Santa Anita Park record for a 6 1/2 furlong horse race on January 22, 2004 in a time of 1:11.13 minutes.
9. Kona Gold set a Santa Anita Park record for a 5 1/2 furlong horse race on January 3, 1999 in a time of 1:01.74 minutes.
10. In the Stock Car division of the National Hot Rod Association National Competition, Larry Hodge won the quarter-mile in a Buick Skylark with a time of 12.738 seconds.
11. In the Super Stock Car division of the National Hot Rod Association National Competition, Jeff Taylor won the quarter-mile in a Pontiac Grand Am with a time of 8.844 seconds.
12. At the Athens Olympics in 2004, Natalie Coughlin (USA) set the 100m backstroke swimming record at 59.68 seconds.
13. At the Stockholm Olympics in 1912, Duke Kahanamoku (USA) set a freestyle swimming record of 1:02.4 for 100 meters.

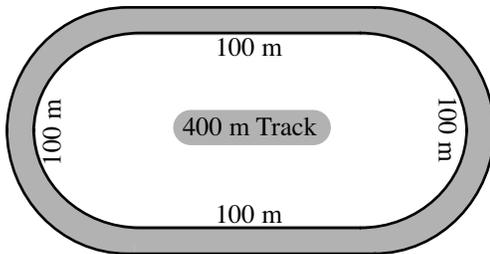
14. In 1967, Michael Johnson (USA) set a world record in the 400 m run of 43:18 seconds.
15. In 1988, Florence Griffith Joyner (USA) set an Olympic record in the 200 m run of 21.34 seconds.
16. Richard DeBernardis set a U.S. bicycling distance record of biking the perimeter of the United States, 12,092 miles in 180 days.
17. In 2004, Lance Armstrong (USA) won his the Tour de France bicycle race with a time of 83:36:02 for 3429 km.

Explore

18. From 1875 to 1896, the Kentucky Derby was run on a race track that was 1 1/2 miles long. The record for that distance was set by Spokane in a time of 2:34.5. Since 1896, the Kentucky Derby is run on a 1 1/4 mile track. The record for that distance was set by Secretariat in 1973 of 1:59.4. How much faster was Secretariat in miles per hour?
19. In the first Indianapolis 500, Ray Harroun in 1911 completed the 500 miles in 6:42:08.00. In the 2002 Indianapolis 500, Helio Castroneves completed the 500 miles in 3:00:10.87. How much faster in mph is the 2002 race car?
20. The world record for the 100 m run is 9.78 seconds set by Tim Montgomery (USA) in 2002. If that pace could be kept up for 1 mile, the absolute minimum time for the mile run would surely be established. What would that time be for the men's mile run?
21. The world record for the woman's 100 m run is 10.49 seconds set by Florence Griffith-Joyner (USA) in 1988. If that pace could be kept up for 1 mile, the absolute minimum time for the mile run would surely be established. What would that time be for the women's mile run?
22. The world record for the woman's 1500 m run is 3:50.46 set by Qu Yunxia (CHN) in 1993. If that pace could be kept up for the marathon (26.2 miles), the absolute minimum time for the marathon would surely be established. What is that time for the women's marathon?
23. The world record for the 1 hour run of 21,101 m was set by Arturo Barrios (MEX) in 1991. If that pace could be kept up for the marathon (26.2 miles), what would be the time for the marathon run?
24. In 1946, greyhound Bah's Choice from England was the first dog to break the 29 second mark in a 525 yard race with a time of 28.99 seconds. The present United States greyhound racing record for a comparable distance, 5/16 mile, was set by Be My Bubba in 2000 of 29.33 sec. Which dog ran faster in miles per hour?

Section 5.4 Race Tracks

Speed and distance come together when racing around a track. Many foot races, horse races, dog races, skating races, automobile races, and bicycle races are held on track that have two semicircular turns and two straight sides. Let us first consider a 400-meter running track which has two straightaways, each 100 meters long, and two equal semicircles measured along the inside rail that are 100 meters long. Such a track is called an **equal-quadrant track**. Suppose that the track has eight lanes, each 1 meter wide.



Example 1

If a runner ran one lap of the track in the second lane, how far did she run?

Solution: You might think that the answer is 400 meters, since it is a 400 meter track.

Both straightaways are 100 meters but as you go further out from the inside rail on the semicircular turns, you run farther. To calculate the distance around the turns, we use the formula for the circumference (C) of a circle with radius (r): $C = 2\pi r$. Since the two semicircles on the inside rail form a circle with a circumference of 200 m, we can find the radius of the inside rail semicircles.

(Note: In this section, 3.14 is used as the value of π .)

$$\begin{aligned} C &= 2\pi r \\ 200 &= 2\pi r \\ \frac{200}{2\pi} &= r \\ 31.83 &\approx r \end{aligned}$$

Now when you run in the second lane, the semicircle turns have a radius that is one meter longer or 32.83 m. Thus, distance around both turns in the second lane is:

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(32.83) \\ &\approx 206.2 \text{ m} \end{aligned}$$

Thus, a lap in the second lane has a total distance of 406.2 m, 200 meters for the straightaways plus 206.2 m for the turns.

Example 2

A runner jogs ten laps of the track in the eighth lane. How far does he run

- measured in meters and
- measured in miles?

Solution:

- a) Let us first determine the distance of one lap in the eighth lane. The straightaways are still 100 m each but the semicircular turns now have a radius that is 8 m more than the radius of the inside rail.

$$r = 31.83 + 8 = 39.83 \text{ m}$$

The distance around both turns in the eighth lane is as follows.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(39.83) \\ &\approx 250.1 \text{ m} \end{aligned}$$

Thus, a lap in the eighth lane is 450.1 m, 200 meters for the straightaways plus 250.1 m for the turns. Ten laps would be ten times that amount or 4501 m.

- b) To change 4501 meters into miles we can set up a proportion comparing miles to meters using the conversion, $6.2 \text{ mi} \approx 10,000 \text{ m}$.

Let $x =$ the distance in miles

$$\begin{aligned} \frac{6.2}{10,000} &= \frac{x}{4501} \\ 10,000x &= 27,906.2 \\ x &\approx 2.8 \end{aligned}$$

By running ten laps in the eighth lane, the jogger ran about 2.8 miles.

Example 3

In the two previous examples, we have seen that the distance you run increases as you are further out from the inside rail. If the lanes are 1 m wide, show that the distance per lap increases by the same amount. Why would this be important in a 400 m race?

Solution: The problem is asking us to determine if the distance in the next lane further away from the rail has increased by the same amount over the lane closer to the rail. This can be answered using some algebra.

Let $r =$ the radius of the turn in any lane

$2\pi r =$ the circumference of the turns in that lane

$r + 1 =$ the radius of the lane that is 1 m further away from the rail

$2\pi(r + 1) =$ the circumference of the turns in that lane

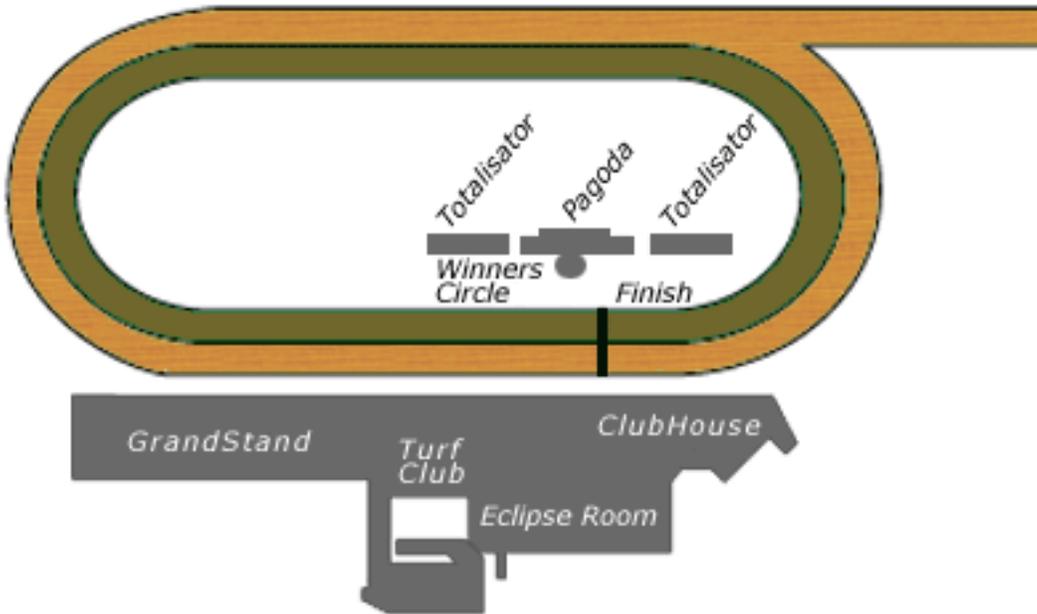
The difference in the distances (d) would be as follows.

$$\begin{aligned} d &= 2\pi(r + 1) - 2\pi r \\ d &= 2\pi r + 2\pi - 2\pi r \\ d &= 2\pi \approx 6.3 \text{ m} \end{aligned}$$

The distance of each lap increases by the same amount, about 6.3 meters. This fact would be important in any race where competitors stay in their lane around the turns.

In a 400 m race, the runners stay in their lane for two turns. To ensure that each runner runs 400 m, the runners would be staggered by 6.3 m. That means the runner in each lane further away from the inside rail would start 6.3 m ahead of the runner in the lane closer to the rail.

In other events such as horse, dog, auto, and skate racing, the tracks are not equal-quadrant tracks. The lengths of the turns and the straightaways are not equal. Let us examine such a track Churchill Downs, Louisville Kentucky, the home of horse racing's Kentucky Derby.



PICTURE OF CHURCHHILL DOWNS TRACK

<http://www.horseparlor.com/churchhill-ky.html>

The track is a one mile oval with straightaways of 1348 feet.

Example 4

- What is the distance around each turn?
- What is the radius of each turn at Churchill Downs?

Solution:

- This oval track consists of two straightaways and two semi-circles which total to 1 mile. The distance around the inside rail of that the track is the length of the two 1348 ft straightaways and a circle.

If r = the radius of the circle, the circumference of the circle $C = 2\pi r$.

$$1348 + 1348 + 2\pi r = 5280$$

$$2696 + 2\pi r = 5280$$

$$2\pi r = 2584$$

The distance of the two turns is $C = 2\pi r = 2584$. So each turn is $\frac{2584}{2} = 1292$ ft.

b) Solving for r in the previous result, we get the following.

$$2\pi r = 2584$$

$$r = \frac{2584}{2\pi} \approx 411.5 \text{ ft}$$

Example 5

If a horse runs 10 feet from the inside rail of the Churchill Downs track for two complete turns, how much further does it run?

Solution: From Example 4, the distance around the inside for two turns is 2584 feet and the radius of the turns is 411.3 ft. If a horse runs 10 ft from the rail it increases the radius of the turns by 10 feet. The distance around two turns can be found using the formula $C = 2\pi r$.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(411.3 + 10) \\ &= 2\pi(421.3) \\ &= 2645.8 \text{ ft} \end{aligned}$$

The increase in distance is $2645.8 - 2584 = 61.8$ ft.

The race tracks discussed so far have been flat tracks. However, in bicycle track racing, the turns and straightaways are banked. The banking increases the distances further from the inside around a turn to a lesser extent than the increase of a flat track. A standard short track cycling oval is 250 m long, with straightaways of 37 m banked at 4° , and turns banked at 22° . Each lap cycled 1 m from the inside of the oval increases the radius of the turns by 0.927 m instead of 1 m.

Example 6

In the 250 m short track cycling oval described above:

- what the distance around the inside of each turn?
- what is the inside radius of the turns?
- what is the distance of a lap cycled 2 meters from the inside of the oval?

Solution:

- A 250 m lap consists of two 37 m straightaways and two turns which form a circle.

Let r = the radius of the turn

$$37 + 37 + 2\pi r = 250$$

$$74 + 2\pi r = 250$$

$$2\pi r = 176$$

The distance around both turns is the circumference of the circle,

$$C = 2\pi r = 176 \text{ m}$$

One turn is one half of that circumference is $\frac{176}{2} = 88$ m.

b) Solving for r in the previous result, we get:

$$2\pi r = 176$$

$$r = \frac{176}{2\pi} \approx 28.0 \text{ m}$$

c) Two meters from the inside the radius increases by twice the increase of the radius as explained before Example 6, that is $2(0.927)$. Thus, the length of the turns can be found using $C = 2\pi r$.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(28 + 2(0.927)) \\ &= 2\pi(29.854) \\ &\approx 187.5 \text{ m} \end{aligned}$$

Thus, the distance for the lap is $37 + 37 + 187.5 = 261.5$ m.

Our study of race tracks has shown that the distance around a turn increases as you move from the inside of the turn. The exact amount of increase can be determined by using mathematics.

§5.4 Explain → Apply → Explore

Explain

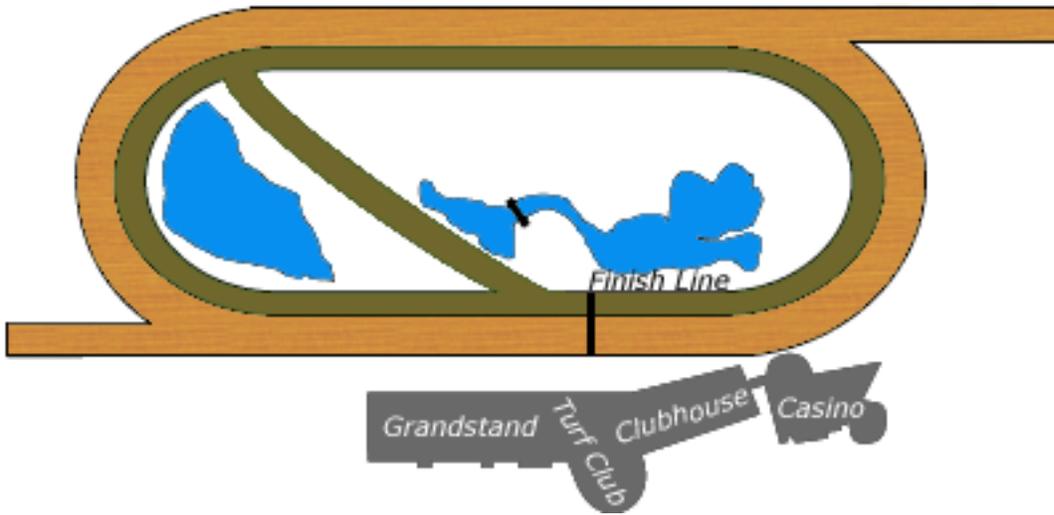
1. What is an equal-quadrant track?
2. In track events in which runners stay in their lanes around a turn, why are the runners in lanes further from the rail given a staggered start?
3. Why do track coaches tell runners not to pass an opponent on the turns?
4. How do you find the circumference of a circle when the radius is known?
5. How do you find the radius of a circle when the circumference of the circle is known?

Apply

6. If the turn in a horse race is a semicircle with a radius of 100 yards, how far is the distance around the turn if the horse runs at the rail?
7. If the turn in a horse race is a semicircle with a radius of 100 yards, how far is the distance around the turn if the horse runs two yards from the rail?
8. If the turn in a horse race is a semicircle with a radius of 100 yards, how far is the distance around the turn if the horse runs 5 yards from the inside rail?
9. If you run 10 laps in the 6th lane of a 440 yard equal-quadrant track that has lanes that are 3.5 feet wide, how many miles do you run?
10. How much further do you run by running a lap in the second lane instead of the first lane of a 440 yard equal-quadrant track that has lanes that are 3.5 feet wide?
11. Determine the amount to stagger each lane for a race in a 440 yard equal-quadrant track that has lanes that are 3.5 feet wide and runners stay in their lanes for a 440 yard race.

12. A 400m Olympic speed skating oval has 111.98 m straightaways and two 4 m wide lanes for head-to-head competition of ice skaters. The radius of the inside turns is 26 m. In a race a skater does one turn in the outside lane and one turn in the inside lane. Show that by doing this a skater does skate 400 m.
13. For the speed skating oval in Problem 12, how far does a skater skate if he does an entire lap in the outside lane?
14. For the speed skating oval in Problem 12, how far does a skater skate if he does an entire lap in the inside lane?

Pictured below is the $1\frac{1}{8}$ mile Hollywood Park Horse racing track in southern California. The straightaways are 1591 feet long.



PICTURE OF HOLLYWOOD PARK

<http://www.horseparlor.com/hollywoodpark-ca.html>

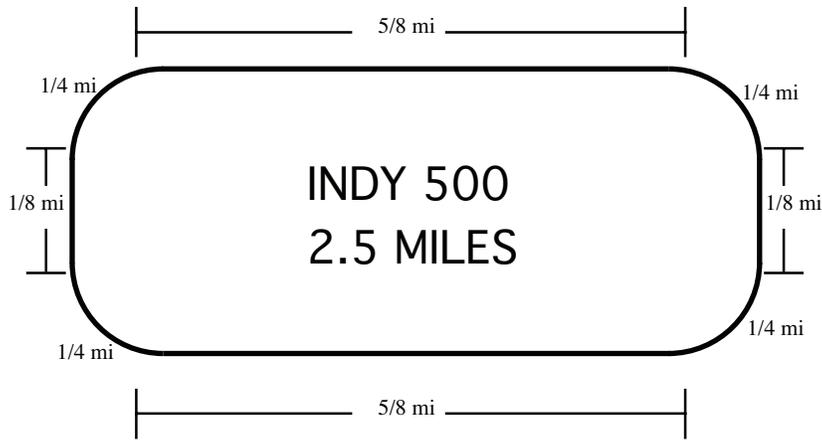
15. What is the radius and length of each turn?
16. How much is the length of a lap increased by if a horse runs the lap 9 feet from the inside rail?
17. How much is the length of a lap increased by if a horse runs the lap 60 feet from the inside rail?

Explore

A long track in bicycle racing is a 400 m oval with 100 m straightaways and two semi-circle turns banked at an angle of 42° . The turns add 0.743 meters per lap for each meter away from the inside rail of the track.

18. What is the distance around each turn at the inside rail?
19. What is the radius of the turns at the inside rail?
20. How far does a biker travel racing an entire lap 4 meters from the inside rail?
21. How far does a biker travel racing a scheduled 800 m race 4 meters from the inside rail?

The Indianapolis 500 Race Track is a 2.5 mile rectangular oval track that consists of two $\frac{5}{8}$ mile straightaways, two $\frac{1}{8}$ mile straightaways, and four quarter-circle turns of $\frac{1}{4}$ miles each. The width of the straightaways are 50 feet.



22. What is the radius of each of the quarter turns?
23. How many miles is a lap driven 20 feet from the inside of the track?
24. How many miles is a lap driven 40 feet from the inside of the track?
25. If you measured the entire 200 laps of the Indy 500 fifty feet from the inside of the track, how long would that be?