Chapter 9 Parametric Equations and Polar Coordinates

Section 9.1 Plane Curves and Parametric Equations

1. Find the parametric equations of a line through the points $(2,3)$ and $(5,1)$.

2. Find the parametric equations of a line through the points $(2,3)$ and $(2,12)$.

3. Find the parametric equations of a circle through the points $(2,3)$ and with center $(1,4)$. The answer should be in the form $x(t) = a + r\cos t, y(t) = b + r\sin t$.

4. Describe the graph of $x(t) = a + r\cos t, y(t) = b + r\sin t$ versus the graph of $x(t) = a + r\cos 6t, y(t) = b + r\sin 6t$ on the interval $0 \le t \le 2\pi$.

5. Graph of $x(t) = t + 0.5\cos 30t, y(t) = t^2 + 0.5\sin 30t$ on the interval $-2 \le t \le 2$. Use $tstep = 0.01$. Explain why this graph could have been predicted.

6. Sketch the graphs of $y = \sin x$ and its inverse without a calculator. Now, use a calculator to graph $y = \arcsin x$. Why is the calculator drawing $y = \arcsin x$ only for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$?

7. Write the parametric equations and the time interval that shows the graph of $y = \arcsin x$ for $-2\pi \le y \le 2\pi$.

8. Use the results of the previous problem to show how to graph the inverse of any function $y = f(x)$ using a calculator and parametric equations. Note that the inverse does not need to be a function.

9. Determine the intersection of the lines

$$\begin{cases} x(t) = 5 - 2t \\ y(t) = 1 + 4t \end{cases} \quad \text{and} \quad \begin{cases} x(t) = 8 - 2t \\ y(t) = 1 + t \end{cases}$$

10. Determine the intersection of the lines

$$\begin{cases} x(t) = 4 + 3t \\ y(t) = 3 + 5t \end{cases} \quad \text{and} \quad \begin{cases} x(t) = 1 + 6t \\ y(t) = -2 + 10t \end{cases}$$

Section 9.2 Calculus and Parametric Equations

1. Sketch the graph of the following equations on the interval $0 \le t \le \pi$.

$$\begin{cases} x(t) = \sin 2t \\ y(t) = 2\sin^2 t \end{cases}$$

   (a) Find the parametric form of the equation of the tangent line to the curve at $t = \frac{\pi}{6}$.
   (b) Find the value(s) of t and the point(s) (x, y) for which the curve has a horizontal tangent line.
   (c) Find the value(s) of t and the point(s) (x, y) for which the curve has a vertical tangent line.

2. Sketch the graph of the following equations on the interval $0 \le t \le 6\pi$.

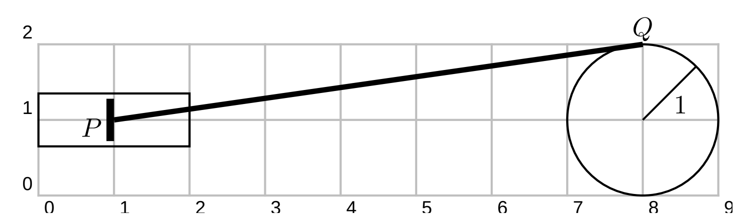
$$\begin{cases} x(t) = t - 1.5\sin t \\ y(t) = 1 - 1.5\cos t \end{cases}$$

   (a) Find the velocity function.
   (b) Determine the intervals when the horizontal component of velocity is negative. What does this mean for a particle traveling along this path?
   (c) At what time(s) t and point(s) (x, y) along the curve does the particle reach its maximum speed?

3. A piston drives a rotating wheel by means of a linkage PQ with a constant length $5\sqrt{2}$ inches. The wheel has radius 1 inch. The motion of point Q, which is connected to the wheel, is given by the equations

$$\begin{cases} x(t) = 8 - \sin t \\ y(t) = 1 + \cos t \end{cases}$$

The point P is on the piston. Find the parametric equations for point P. Compare the speed of P with the speed of Q.



Section 9.3 Arc Length and Surface Area in Parametric Equations

1. Determine the arc length of the the following equations on the interval $0 \le t \le 2\pi$.

$$\begin{cases} x(t) = t - \sin t \\ y(t) = 1 - \cos t \end{cases}$$

2. Determine the surface area when the above curve is rotated around the x axis.
3. What is the relationship between arc length and speed?

Section 9.4 Polar Coordinates

1. Use a piece of polar graphing paper to sketch by hand an *accurate* graph of $r = 10\sin 3\theta$.
2. Convert the equation $x^2 + (y-2)^2 = 4$ into polar form. Write the solution as $r = f(\theta)$.
3. Convert the equation $Ax + By = C$ into polar form. Write the solution as $r = f(\theta)$.
4. Sketch the graph of $r = 8\cos 2\theta$ and $r = 8\cos 2(\theta - \frac{\pi}{12})$. What is the relationship between the graphs?
5. What is the relationship between the graph of the polar function $r = 4\cos 2\theta$ and the graph of the parametric equations $x = 1 + 4\cos 2t\cos t,\ y = 2 + 4\cos 2t \sin t$?
6. Use the results of the previous problem to explain how to graph any polar function $r = f(\theta)$, center at a point (h,k) rather than at the origin.
7. Create a polar graph than looks better than the graph done by anyone else in the class.

Section 9.5 Calculus and Polar Coordinates

1. Find the polar coordinate form of the equation of a line that passes through the origin and has slope $\frac{2}{3}$.
2. Find the equation of the tangent line to $r = 10\sin 3\theta$ when the graph passes through the origin.
3. Find the equation of the tangent line to $r = 10\sin 3\theta$ when $\theta = \frac{\pi}{6}$.
4. Find the area inside of $r = 10\sin 3\theta$.
5. Find the area inside the smaller loop of $r = 2 + 3\cos\theta$.
6. Find the area inside of $r = 10\sin 3\theta$ and outside of $r = 6$.
7. Find the arc length of the curve $r = 2a\cos\theta$.
8. Assuming a and b are positive constants, find the arc length of the curve $r = \dfrac{ab}{b\cos\theta + a\sin\theta}$ on the interval $0 \le \theta \le \dfrac{\pi}{2}$.

Section 9.6 Conic Sections

1. Find all points that are equidistant from $(2,3)$ and the line $y = 5$.
2. Find all points that are twice as far from the point $(0,2)$ as from the line $y = -2$.
3. Find all points that are twice as far from the point $(0,2)$ as from the point $(-2,0)$.
4. A parabolic dish has a diameter of 6 meters and a depth of 1 meter. Where is the focal point of the dish?
5. Jack and Jill are twelve kilometers apart along a north-south coastline. Jack receives a radio signal 0.03 seconds before Jill receives the same signal. If radio signals travel at 3×10^8 m/sec, determine the possible points that could be the origin of the radio signal.