Chapter 6 Integration Techniques

Section 6.1 Review of Formulas and Techniques

1. Perform the following integrations without the use of a table or machine. Use of the integration card is acceptable.

   (a) $\int \sin 3x\,dx$

   (b) $\int \tan(2x-3)\,dx$

   (c) $\int (2x-1)^2\,dx$

   (d) $\int (2x-1)^{10}\,dx$

   (e) $\int \sqrt{(2x-1)}\,dx$

   (f) $\int \dfrac{2x+1}{x}\,dx$

   (g) $\int \dfrac{x}{2x+1}\,dx$

   (h) $\int \dfrac{\ln 3x}{x}\,dx$

   (i) $\int \dfrac{1}{3x\ln x}\,dx$

   (j) $\int \dfrac{\arcsin x}{\sqrt{1-x^2}}\,dx$

   (k) $\int \dfrac{\arcsin 2x}{\sqrt{1-4x^2}}\,dx$

   (l) $\int \dfrac{x\arcsin 2x^2}{\sqrt{1-4x^4}}\,dx$

2. Sketch a graph of $y=\sqrt{4-x^2}$ on the interval $-2\le x\le 2$. Use your graph to determine the value of $\int_{-2}^{2}\sqrt{4-x^2}\,dx$.

3. Sketch a graph of $y=x\sqrt{16-x^4}$ on the interval $-2\le x\le 2$. Use your graph to determine the value of $\int_{-2}^{2} x\sqrt{16-x^4}\,dx$.

Section 6.2 Integration by Parts

Perform the following integrations without the use of a table or machine. Use of the integration card is acceptable. Integration by parts is not needed for all problems.

1. $\int x\cos x\,dx$
2. $\int x\cos 5x\,dx$
3. $\int x^2\cos 5x\,dx$
4. $\int \dfrac{x}{\cos^2 x}\,dx$
5. $\int \arctan(2x)\,dx$
6. $\int x\ln(3x)\,dx$
7. $\int \sec^3 x\,dx$
8. $\int e^{-3x}\cos(4x)\,dx$
9. $\int x\sqrt{2x-1}\,dx$
10. $\int xe^{-x^2}\,dx$
11. $\int x^3e^{-x^2}\,dx$
12. $\int \dfrac{x\arcsin(x)}{\sqrt{1-x^2}}\,dx$

Section 6.3 Trigonometric Techniques of Integration

Perform the following integrations without the use of a table or machine. Use of the integration card is acceptable.

1. $\int \cos^2 4x\,dx$
2. $\int \cos^3 4x\,dx$
3. $\int \tan^2 4x\,dx$
4. $\int \tan^3 4x\,dx$
5. $\int \sec^2 4x-\tan^2 4x\,dx$
6. $\int x^2\cos 5x\,dx$
7. $\int \sqrt{4-9x^2}\,dx$
8. $\int \sqrt{9x^2+4}\,dx$
9. $\int \dfrac{x^2}{\sqrt{x^2-1}}\,dx$
10. $\int \dfrac{x}{\sqrt{x^2-1}}\,dx$
11. $\int \sqrt{1+\cos(3x)}\,dx$
12. $\int \sqrt{1+e^{3x}}\,dx$
13. $\int \dfrac{d\gamma}{\sqrt{\gamma^2-\psi^2}}$
14. $\int \dfrac{d\psi}{\sqrt{\gamma^2-\psi^2}}$

Section 6.4 Integration of Rational Functions Using Partial Fractions

1. Perform the following integrations without the use of a table or machine. Use of the integration card is acceptable.

   (a) $\int \frac{3x}{x^2+4}\,dx$

   (b) $\int \frac{3}{x^2-4}\,dx$

   (c) $\int \frac{5}{x^2+6x+5}\,dx$

   (d) $\int \frac{3x}{x^2+6x+7}\,dx$

   (e) $\int \frac{3x^2}{x^2-4}\,dx$

   (f) $\int \frac{x}{x^3-3x^2+3x-1}\,dx$

   (g) $\int \frac{5}{(x-1)^2(x+4)}\,dx$

   (h) $\int \frac{1}{x^4-16}\,dx$

2. As a challenge, $\int \frac{1}{x^4+4}\,dx$ (Hint: $x^4+4 = x^4+4x^2+4-4x^2$)

3. As a challenge, $\int \sqrt{\tan x}\,dx$

Section 6.5 Integration Tables and Computer Algebra Systems

1. Use a computer algebra system to find

$$\int \frac{dx}{3\cos x + 4\sin x}$$

2. As a challenge, do the above problem by hand. Hint: Let $u = \tan\frac{x}{2}$.

3. *Mathematica* produces

$$\int \frac{1}{\sqrt{-x^2-4x}}\,dx = \frac{2\sqrt{x}\sqrt{4+x}\,\operatorname{arcsinh}\left(\frac{\sqrt{x}}{2}\right)}{\sqrt{-x(4+x)}} + C$$

   Perform the integration by hand and determine if your result is equivalent to *Mathematica's*.

4. Work on a bunch of problems from the class handout. Check your answers using a machine.

Section 6.6 Improper Integrals

1. $\int_{-1}^{2} \frac{dx}{x^{2/5}}$

2. $\int_{1}^{2} \frac{dx}{(1-x)^2}$

3. $\int_{0}^{\infty} x^2 e^{-5x}\,dx$

Chapter 7 First Order Differential Equations

1. What is the difference between an initial value problem (IVP) and a differential equation?

2. Solve $y' = x^2 + e^x, y(0) = 3$

3. Solve $y' = 3y^2, y(1) = 3$

4. Solve $y' = x^2 y, y(1) = 3$

5. The rate of decay of a radioactive isotope is proportional to the amount of the isotope. The half-life of the isotope Cesium 134 is 2.1 years. If you start with 5 grams of Cesium 134

   (a) Write the model describing the amount of Cesium 134 at any time t.

   (b) How much Cesium 134 do you have after 1 week?

6. The temperature of a hot liquid will cool at a rate proportional to the difference between temperature of the liquid and the room temperature, M. If the initial temperature of the liquid is T_0, start with the initial value problem for this model and derive the equation for he temperature $T(t)$ of the liquid at any time t.

7. There is a virus moving through a school of 1500 children. If the rate of change in the number of children who have the virus is proportional to the number of children who do not have the virus, there was one child who had the virus on the first day and 58 who had the virus on the third day, determine when 1400 of the children will have the virus. Let $t = 0$ be day 1.