

## Convergence Tests

Name	Series	Converges	Diverges	Other
$k$ th term (Divergence)	$\sum_{k=1}^{\infty} a_k$		$\lim_{k \rightarrow \infty} a_k \neq 0$	cannot be used for convergence
geometric	$\sum_{k=0}^{\infty} ar^k$	$ r  < 1$	$ r  \geq 1$	$S = \frac{a}{1-r}$
telescoping	$\sum_{k=1}^{\infty} (a_k - a_{k-1})$	$\lim_{k \rightarrow \infty} S_k = L$		$S$ can be found by expanding
$p$ -series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	$p > 1$	$p \leq 1$	$0 < R_N < \frac{N^{1-p}}{p-1}$
alternating	$\sum_{k=1}^{\infty} (-1)^k a_k$	$a_k$ decrease, approach 0		$ R_N  < a_{N+1}$
integral	$\sum_{k=1}^{\infty} a_k, \quad a_k = f(k)$	$\int_1^{\infty} f(x) dx$ converges	$\int_1^{\infty} f(x) dx$ diverges	$0 < R_N < \int_N^{\infty} f(x) dx$
ratio	$\sum_{k=1}^{\infty} a_k$	$\lim_{k \rightarrow \infty} \left  \frac{a_{k+1}}{a_k} \right  < 1$	$\lim_{k \rightarrow \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1$	
root	$\sum_{k=1}^{\infty} a_k$	$\lim_{k \rightarrow \infty} \sqrt[k]{ a_k } < 1$	$\lim_{k \rightarrow \infty} \sqrt[k]{ a_k } > 1$	
direct comparison	$\sum_{k=1}^{\infty} a_k$	$0 < a_k < b_k$ and $\sum_{k=1}^{\infty} b_k$ converges	$0 < b_k < a_k$ and $\sum_{k=1}^{\infty} b_k$ diverges	
limit comparison	$\sum_{k=1}^{\infty} a_k$	$\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = L > 0$ and $\sum_{k=1}^{\infty} b_k$ converges	$\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = L > 0$ and $\sum_{k=1}^{\infty} b_k$ diverges	$L$ must be a finite number.

## Series

Function	Series	Closed Form	Interval of convergence
$\frac{1}{u}$	$1 - (u - 1) + (u - 1)^2 - (u - 1)^3 + \dots$	$\sum_{k=0}^{\infty} (-1)^k (u - 1)^k$	$0 < u < 2$
$\frac{1}{1 - u}$	$1 + u + u^2 + u^3 + \dots$	$\sum_{k=0}^{\infty} u^k$	$-1 < u < 1$
$\ln u$	$(u - 1) - \frac{(u - 1)^2}{2} + \frac{(u - 1)^3}{3} \dots$	$\sum_{k=1}^{\infty} \frac{(-1)^{k-1} (u - 1)^k}{k}$	$0 < u \leq 2$
$e^u$	$1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \dots$	$\sum_{k=0}^{\infty} \frac{u^k}{k!}$	$-\infty < u < \infty$
$\cos u$	$1 - \frac{u^2}{2!} + \frac{u^4}{4!} - \frac{u^6}{6!} + \dots$	$\sum_{k=0}^{\infty} \frac{(-1)^k u^{2k}}{(2k)!}$	$-\infty < u < \infty$
$\sin u$	$u - \frac{u^3}{3!} + \frac{u^5}{5!} - \frac{u^7}{7!} + \dots$	$\sum_{k=0}^{\infty} \frac{(-1)^k u^{2k+1}}{(2k+1)!}$	$-\infty < u < \infty$
$\arctan u$	$u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} \dots$	$\sum_{k=0}^{\infty} \frac{(-1)^k u^{2k+1}}{2k+1}$	$-1 \leq u \leq 1$
$\arcsin u$	$u + \frac{u^3}{2 \cdot 3} + \frac{1 \cdot 3u^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5u^7}{2 \cdot 4 \cdot 6 \cdot 7} + \dots$	$\sum_{k=0}^{\infty} \frac{(2k)! u^{2k+1}}{(2^k k!)^2 (2k+1)}$	$-1 \leq u \leq 1$
$(1 + u)^k$	$1 + ku + \frac{k(k-1)u^2}{2!} + \frac{k(k-1)(k-2)u^3}{3!} + \dots$		$-1 < u < 1$
$(1 + u)^{-k}$	$1 - ku + \frac{k(k+1)u^2}{2!} - \frac{k(k+1)(k+2)u^3}{3!} + \dots$		$-1 < u < 1$