Measurement

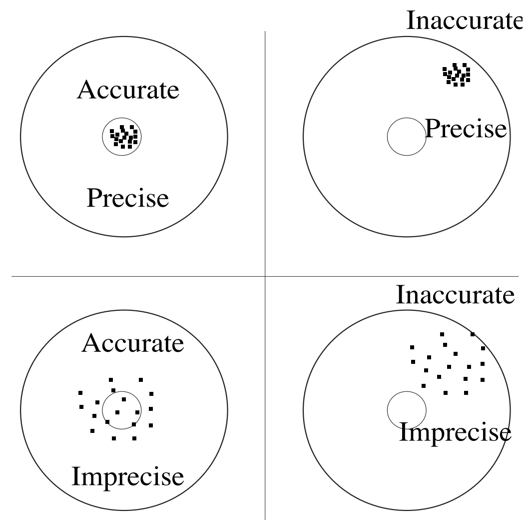
INTRODUCTION

Measurements are made with tools. The tool can be as simple as a ruler or as complex as the Hubble Space Telescope. It is typical that the finer the measurement required, the more expensive the tool becomes which must be used to make the measurement. For example, we have two types of electronic balances in this laboratory. One is a decigram balance, and the other is a milligram balance. The first is capable of measuring to the nearest 0.1 g, while the other will measure to the nearest 0.001 g. The first costs about $125, the other costs about $1000.

A general principal with regard to making measurements and recording the results is: ***Always record the measurement to the full capacity of the tool used. Never record the measurement with more than the capacity of the tool used.*** For example: If you were to weigh a penny on a decigram balance, you might see *2.4 g* on the display. If you used an electronic milligram balance, you might see something like *2.413 g* for that same penny appearing in the display window. If you see a display given to the nearest milligram, ***then record the mass to that level of accuracy.*** It might be that the display showed *2.400 g* in the display window. Then record 2.400 g on the data sheet. The manner in which the data is recorded indicates the accuracy of the tool used to make the measurement. If you record 2.4, it is assumed that the balance used can only give readings to the tenth place. If, in fact, you used a balance that shows 2.400 g, make sure you write 2.400 as the recorded mass.

(Note: It is a common beginning student error to only record the first few numbers appearing on the display, and then to fill in zeroes when the instructor reminds the class of the need to record to the nearest milligram. A series of measurements that all end in zero is highly unlikely, unless there has been student carelessness involved.)

Balances can give readings different from the true value if they lose their calibration. If the same object is weighed a series of times on a milligram balance, and a series of times on a decigram balance, the results could look like this: (each dot represents one of the weighings)



The top drawings are for the milligram balance. The right hand drawings would be for balances out of calibration. Be familiar with the way the words *accurate* and *precise* are used.

Significant Digits: When making a measurement, the last digit always involves some estimation. You will notice on the electronic balance that the last digit will sometimes flicker back and forth. There are rules for working with numbers obtained from measurements that take this estimation into account and preserve us from making foolish calculations. An example of such a calculation is that of the tour guide in Egypt who told visitors that one of the pyramids was 5006 years old. When asked how he knew that, he replied "When I first came to work 6 years ago, I was told that it was 5000 years old!" Of course, his error lies in adding an accurate 6 years to a roughly estimated 5000 years. Your textbook gives the rules for significant digits and many examples of their use. Here is a summary of the rules.

1. In numbers without zeroes, the rule for counting significant digits (sd) is simple: the number of significant digits equals the number of digits. So, 63.44 has four sd. 25,444 has five sd.
2. Zeroes in the midst of other digits are always significant. So, 6.023 has four sd.

3. For numbers with decimals and preceding or trailing zeroes, counting sd's requires more care. Preceding, place-holding zeroes are not significant. So, 0.00345 has three sd (not 5 or 6). Writing such numbers using scientific notation makes this clear. 3.45×10^{-3} clearly has three sd. Trailing zeroes are always significant. So, 0.0034500 has five sd. This number would be written 3.4500×10^{-3} in scientific notation.

4. Zeroes at the end of a number without a decimal cause the most trouble. The number 1010 has 3 sd. If it is written with a decimal following the last zero, 1010., it has 4 sd. If the number 1000 is good to 3 sd, it will be written as $10\bar{0}0$, or sometimes as 1000 ± 10. Remember, the reason a number is good to any amount of sd lies in the tool used to measure it.

5. When multiplying or dividing, the number with fewer significant digits determines the amount of significant digits in the result. So, 13.1 x 14.566 = 191. Your calculator will show 190.8146, and you would round it out to 3 sd. If you are performing consecutive calculations, don't round off until you reach the final number.

6. When adding or subtracting, line up the decimal points, and then, looking at each number from left to right, pick out the column with the number that has the shortest range of significant digits. That is, for the numbers 4340 and 5126, the first number's sd ends in the 10's column, the second number's sd ends in the 1's column. For these two numbers, the 10's column contains the final sd. Your answer cannot continue past the ten's column with significant digits.

$$\begin{array}{r} 4340 \\ \underline{+5126} \\ 9470 \end{array}$$

You see that the sum is rounded to the 10's column. Remember the Egyptian guide adding 5000 and 6. He should have points taken off of his grade for not rounding properly. Here is another example:

$$\begin{array}{r} 12.3402 \\ \underline{+6.22} \\ 18.56 \end{array}$$

Since the second number ends in the hundredth's column, the 02 ending of the first number is rounded off. Notice that this is considerably more complicated than multiplication or division. The answer has 4 sd, while the two numbers used had 6 and 3 sd. The rules of rounding should be followed consistently, and rounding should be done after a series of steps, not for each step. Some calculators can be set to follow the degree of accuracy of the measurements used. Consult your calculator's manual.

7. With logarithms, only the number of digits following the decimal count as significant digits. For example, log(5687) would be written 3.7549. The reason for this becomes evident if we show the number in scientific notation. Then, $\log(5.687 \times 10^3)$ becomes 3.7549, where the 3 is derived from the exponent on ten.

EXPERIMENT

To get hands-on experience with measuring, and with the balances, you will weigh pennies. Work with a partner. Pick up a packet of pennies. There will be four pennies dated 1981 and earlier, and four pennies dated 1983 and later. Separate the pennies into two groups, one group the 1981 and earlier, the other the 1983 up to the present. One partner takes one set of pennies, the other takes the other set. Each of you will weigh each penny in your set, and then all four pennies together, using a milligram balance. Record all weights to the third decimal place (0.001). Consult the *Using Balances* discussion on the web site for instructions on how to zero and use these balances. Record the date and mass of each penny in the data table on the next page. Next, each of you will weigh each penny on a decigram balance, recording the weights to the first decimal place (0.1). Follow the instructions in the data table for analyzing your data. Repackage the pennies and return them. The pennies from 1981 and earlier are made of a copper alloy. Those from 1983 and later are copper plated zinc. Pennies minted in 1982 may be of either composition

You will see that the range of values obtained for the mass of the older pennies is small. The newer pennies also have a small range. The range of values is small enough, in fact, that weighing a penny will allow you to predict its time of minting.

The ranges of values for the masses of some objects is large. Do all watermelons weigh almost exactly the same? How much does the largest man weigh? Is his weight just a fraction of a pound different than the average weight of adult males? No, of course not. Humans are not stamped out by machines, like pennies, nor are watermelons. The deviation from the average weight of an adult male and the weight of an individual adult male can be and usually is large.

Given the weight of an object, predictions can be made about the object if you know the range to expect for the weight of the object. If you know that you have either an old penny or a new penny, you can tell which it is if you know the weight. You are comparing an average weight of 2.421 ± 0.020 g to an average weight of 3.034 ± 0.030 g. If you have either an apple or an orange, the weight will tell you nothing about what you have, because the range of weights of oranges and the range of weights of apples crisscrosses. You are comparing an average weight of 150 ± 70 g to an average weight of 210 ± 90 g. (These are guessed average masses---do not make bets based on them.)

Your Data			Partner's Data		
Date of the penny	Mass of the penny (to nearest mg)	Deviation from ave. mass (+ or -) (ave mass 6 rows below)	Date of the penny	Mass of the penny (to nearest mg)	Deviation from ave. mass (+ or -)
	g	g		g	g
	g	g		g	g
	g	g		g	g
	g	g		g	g
Mass of 4 pennies weighed together		g	Mass of 4 pennies weighed together		g
Average mass		g	Average mass		g

Sum of individually weighed pennies	g
Difference, weighed together and sum from individual weights	g
If the difference is more than 0.005g, you may be using the balance incorrectly. See the Instructor.	
Sum of deviation from average mass	g
If the sum is more than 0.003, you may have neglected to use + and - numbers	

Mass of pennies on decigram balance			
Your penny, Date	mass (to nearest dg)	Partner, date	mass (to nearest dg)
	g		g
	g		g
	g		g
	g		g
average	g	average	g

QUESTIONS

1. The mass, on average, of a male is greater than the mass, on average, of a female. Given the mass of a person, could you say for sure whether the person was male or female? Explain.

2. Given the mass of a penny, could you say for sure if it was a copper alloy penny or a copper-clad zinc penny? Explain.

3. Subtract the average mass of the copper-clad zinc penny from the average mass of the copper alloy penny, using the milligram data. How many significant digits are in the answer?

4. Subtract the average mass of the copper-clad zinc penny from the average mass of the copper alloy penny, using the decigram data. How many significant digits are in the answer?

5. Define decigram balance and milligram balance.

6. Which pennies weigh more, the primarily copper ones or the primarily zinc ones? Does it appear that atomic weight is the only consideration for the density of a material, or must there be more to it than that?

Name____ Grade___________ Date ___________

EXERCISES

1. How many significant digits are in each of the following numbers?

45.8736	__________	0.00239	__________	48,000	__________
93.00	__________	3.982×10^{6}	__________	1.70×10^{-4}	__________
0.00590	__________	1.00040	__________	4.00000	__________
3800	__________	23000 ± 10	__________		

2. Round each of the following numbers to 3 significant figures. Do not change their values (that is, if a number is in the thousands before rounding, it will be in the thousands after rounding).

1367	__________	0.0037421	__________	1.5587	__________
12.85	__________	1.6683×10^{-4}	__________	1.632257	__________

3. Perform each of the following indicated operations and give the answer to the proper number of significant digits. Watch the order of operation when both addition/subtraction and multiplication/division are involved in the same problem. When adding or subtracting numbers written in scientific notation, it often helps to rewrite the numbers so each one has the same index (that is, power of ten). For example, to add 3.42×10^{-3} and 5.223×10^{-4}, first rewrite the second number as 0.5223×10^{-3}. Remember, if you make the number smaller, you make the power bigger, and vice versa. 0.5223 is smaller than 5.223, and 10^{-3} is bigger than 10^{-4}.

32.27×1.54	____________	$3.68/0.07925$	____________
1.750×0.0342	____________	$0.00957/2.9465$	____________
$(3.2650 \times 10^{24}) \times (4.85 \times 10^{3})$	____________	$7.56 + 0.153$	____________
$8.2198 - 5.32$	____________	$10.052 - 9.8742$	____________
$(6.75 \times 10^{-8}) + (5.43 \times 10^{-7})$	____________	$0.01953 + (7.32 \times 10^{-3})$	____________
$(8.52 + 4.1586) \times (18.73 + 153.2)$	____________	$(8.52 \times 4.1586) + (18.73 \times 153.2)$	____________
$\sqrt{8.32 \times 10^{-3}}$	____________	$(3.84 \times 10^{-2})^{3}$	____________
$\dfrac{(0.000738 - (8.3 \times 10^{-5}))}{6.298 \times 10^{-8}}$	____________	$\log(22.6)$	____________
		$\ln(12.55)$	____________